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Algorithm for Unfolding Current from Faraday Rotation Measurement

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Abstract

Various methods are described to translate Faraday rotation measurements into a useful representation of the dynamic current under investigation[1]. For some experiments, simply counting the “fringes” up to the turnaround point in the recorded Faraday rotation signal is sufficient in determining the peak current within some allowable fringe uncertainty. For many other experiments, a higher demand for unfolding the entire dynamic current profile is required. In such cases, investigators often rely extensively on user interaction on the Faraday rotation data by visually observing the data and making logical decisions on what appears to be turnaround points and/or inflections in the signal. After determining extrema, inflection points, and locations, a piece-wise, ΔI/Δt, representation of the current may be revealed with the proviso of having a reliable Verdet constant of the Faraday fiber or medium and time location for each occurring fringe.

In this paper, a unique software program is reported which automatically decodes the Faraday rotation signal into a time-dependent current representation. System parameters such as the Faraday fiber’s Verdet constant and number of loops in the sensor are the only user-interface inputs. The central aspect of the algorithm utilizes a short-time Fourier transform (STFT) which reveals much of the Faraday rotation’s hidden detail necessary for unfolding the dynamic current measurement.

I. INTRODUCTION

The pulsed power community and power generation industry is finding increased applications utilizing Faraday effect sensors. The Faraday effect in single-mode fibers permit fast responding current sensing on high-voltage, high-current transmission lines[2][3][4]. The underlying theory of how the Faraday effect works is best described in the quantum mechanical realm, but can be understood on a basic classical electrodynamics level[5]. Various Faraday current sensing configurations along with subtle Faraday effects which may adversely affect the system are described elsewhere[6]. Linear polarized light launched into a Faraday current fiber emerges with a rotating linear polarization state at an angular frequency proportional to dI/dt. The axis of polarization of this emerging light is usually preadjusted through a half-wave plate and then split into two differently oriented analyzers such as the Wollaston beam splitter which conveniently splits the beam and orients the polarization orthogonal to each other. The emerging light out of each facet or analyzer is optically modulated and captured via typical optical-electrical photodetectors and digitizers. The orientation of the analyzers need not be orthogonal. It may be convenient to orient the analyzers at 0° and 45° relative to horizontal to yield optically modulated signals proportional to $\mathrm{Sin}^2[\theta(t)+\phi]$ and $\mathrm{Cos}^2[\theta(t)+\phi]$ (or ½{1-Cos(2[θ(t)+ϕ])} and ½{1+Cos(2[θ(t)+ϕ])} respectively) where ϕ is some initial starting phase. One basic post processing technique is to take the square-root of the ratio of such signals to yield either Tan[θ(t)+ϕ] or Cot[θ(t)+ϕ] signature wave form which indicate increases or decreases in θ(t). The current, I(t), is directly proportional to θ(t) through the simple closed-form equation (1)

$$\theta(t) = \mu_o VBL = V'N'I(t) \qquad (1)$$

where B(t) is the time-dependent magnetic field produced from the dynamic current, I(t), along the closed path, L as defined in Ampere’s law for typical cylindrical symmetric systems. The parameter, N′, is a generic amplifying factor that is multiplicative for both the number of electrical windings and/or loops of Faraday fiber in the sensing region. The Verdet constant, V, is redefined as V′ to absorb the factor μ_o and to be expressed in more convenient units of radians/amps or radians/mega-amps. The resolution of the uncovered current signal, I(t), is a function of the Verdet constant, I(t), and ΔI(t)/Δt. The rotation, θ(t), is proportional to both V′ and I(t) while the time step between each data point is inversely proportional to ΔI(t)/Δt.

This work reports a unique method which utilizes a STFT which uncovers detail particularly during slow-varying signals and using only a fraction of the optical components. The emerging rotating linear polarized light is not split and analyzed directly thus bypassing the half-wave plates, beam splitters, polarizers, and possibly other associated and complementary optical components.

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Needless to say, this translates to tremendous cost savings (more than several thousands of dollars), especially during explosive driven pulsed power application in which most equipment is likely to be expended.

II. DETAILED DESCRIPTION

Traditional method basics

Before detailing the novel method of deciphering a Faraday signal into a useful current representation, a review of unfolding current from a Faraday signal via traditional methods may be useful. As an example, a simulated current, Figure 1, produces a pair of Faraday signals which are optically decoded modulated signals proportional to ½{1-Cos(2[θ(t)+ϕ])} and ½{1+Cos(2[θ(t)+ϕ])} respectively, Figures 2 and 3. The angle, ϕ, is some initial starting phase dependent on the half-wave plate and the state of polarization of the Faraday fiber itself. For clarity, Figure 4 shows a close-up of the two superimposed Faraday signals in a particular region of interest.

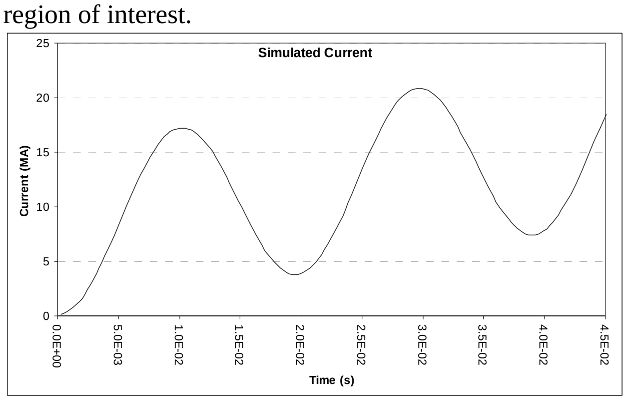

Figure 1. Computer simulated current.

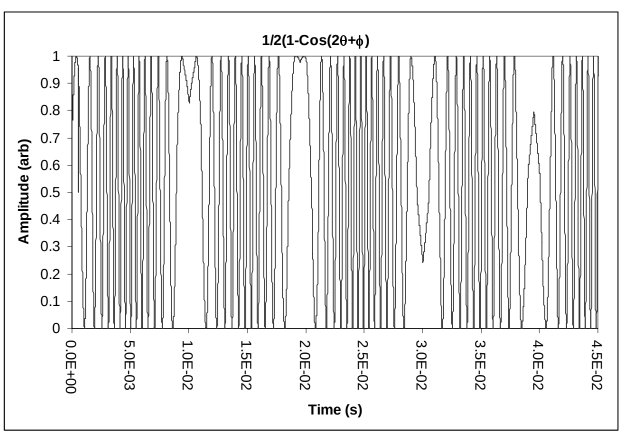

Figure 2. Simulated Faraday decoded signal proportional to ½{1-Cos(2[θ(t)+ϕ])}.

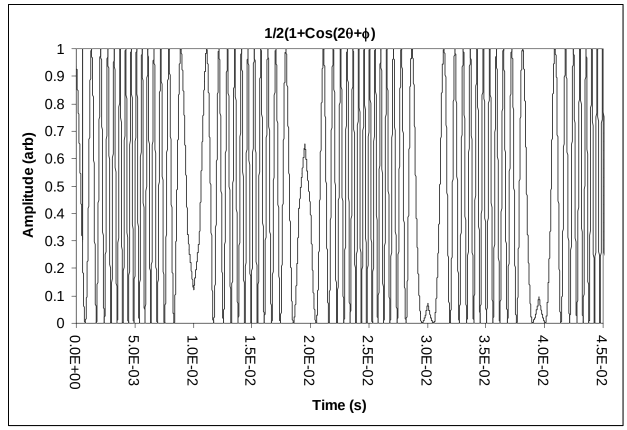

Figure 3. Simulated Faraday decoded signal proportional to ½{1+Cos(2[θ(t)+ϕ])}.

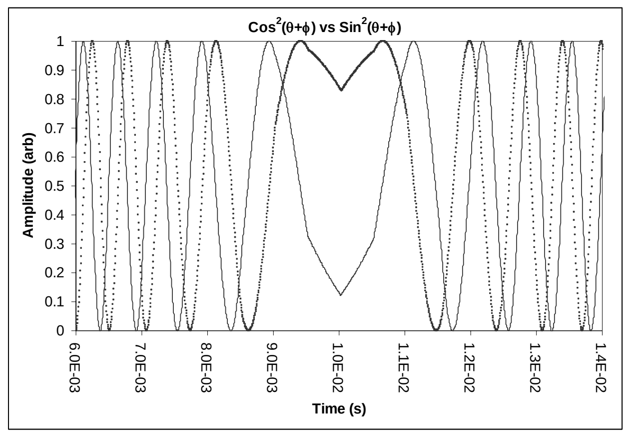

Figure 4. Windowed region of interest of superimposed simulated Faraday signals to show leading and lagging aspects before and after a "turn around" point. This point coincides with a local extrema in the current representation.

Note the qualitative nature of how one signal leads the other before the turnaround and then reverses (lags) itself past the turnaround point. These are all tell signs of a current signal at its local extrema value. A piece-wise current representation may now be reconstructed from these observed leading/lagging crossing points. Starting with equation (1), current I(t) can be expressed in terms of θ(t) as follows:

$$\theta(t) = N'V'I(t) \tag{2}$$

$$I(t) = \frac{\theta(t)}{N'V'} \tag{3}$$

But I(t) can be expressed as a sum of $\Delta I(t_n)$ where t_n is the time at which signal A either leads or lags signal B at its zero crossing. $\Delta I(t_n)$ is further defined as follows:

$$\Delta I(t_n) = \frac{\Delta\theta(t_n)}{N'V'} = \frac{\Delta\theta(t_n)}{\Delta t_n}\frac{\Delta t_n}{N'V'}. \tag{4}$$

In a time Δt_n, an observed Faraday signal has undergone a 2π radian rotation or one full revolution (aka "fringe"). The trigonometric double angle in the observed signals requires an additional division by two to get back θ. Thus, $\Delta I(t_n)$ is

$$\Delta I(t_n) = \left(\frac{2\pi}{2N'V'\Delta t_n} \right) \Delta t_n = \left(\frac{\pi}{N'V'\Delta t_n} \right) \Delta t_n \quad (5)$$

where $\pi/V'\Delta t_n$ is the piece-wise $\Delta I/\Delta t_n$ segment. The sum of each of these $\Delta I/\Delta t_n$ segments times each adjacent Δt_n (Equation 5) represents the total piece-wise reconstructed current shown in Figure 5.

$$I(t) = \sum_n^N \left(\frac{\Delta I}{\Delta t_n} \right) \Delta t_n = \sum_n^N \left(\frac{\pi}{N'V'\Delta t_n} \right) \Delta t_n \qquad (6)$$

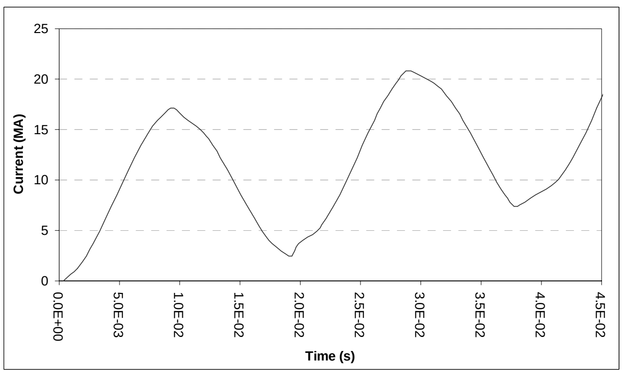

Figure 5. Piece-wise reconstructed current representation from a pair of Faraday optically decoded modulated signals.

Alternate method

Starting with Equation 5, I(t) can also be expressed in terms of frequency, ν, as

$$\Delta I(t_n) = \frac{\Delta\theta(t_n)}{N'V'} = \frac{2\pi}{N'V'} \left(\upsilon_n \Delta t_n \right) \qquad (7)$$

The frequency, ν_n, is determined by a Buneman frequency estimator algorithm[5] over a STFT whose window type and time length is user defined. Figure 6 shows the frequency response with respect to time of Faraday signal shown in Figure 2. In this representation, the local extrema, which reflects both maximas and minimas in the current, are clearly shown where the frequency, ν, goes to zero (i.e. $\Delta\theta/\Delta t = 0$). Likewise, the inflection points are represented by the maximum frequency values. For each successive extrema pair (e.g., maxima to minima or minima to maxima) in time, there is an intermediate inflection point whose sign alternates. By programmatically changing the sign of every other inflection group, the converted frequency response shows both positive and negative frequency (increasing and decreasing, respectively) results shown in Figure 7. Integrating this corrected representation of the Faraday frequency response corresponds precisely to the Faraday rotation angle changing direction through local extrema in the current representation. The unfolded current is revealed by multiplying by the constant, $\pi/N'V'$, shown in Figure 8. Δt_n is no longer limited by the piece-wise "time between fringes." The time difference, Δt_n, can now be reduced to the time resolution of the recording instruments.

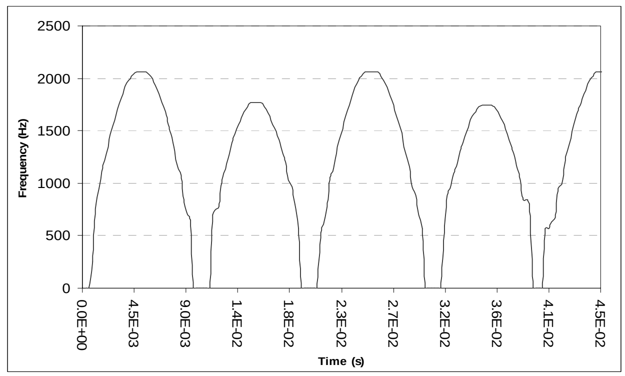

Figure 6. Faraday signal's uncorrected frequency response with respect to time.

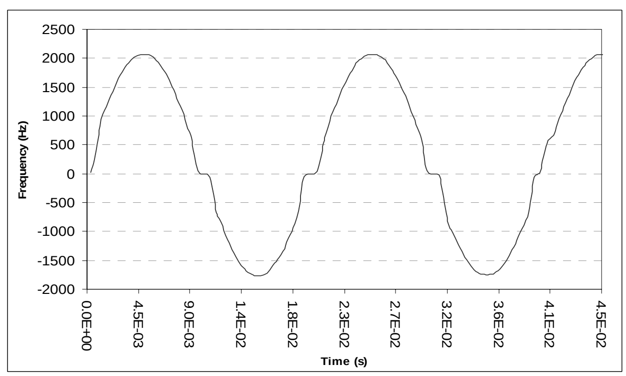

Figure 7. Faraday signal's converted frequency response with respect to time which is directly proportional to $\Delta\theta/\Delta t$.

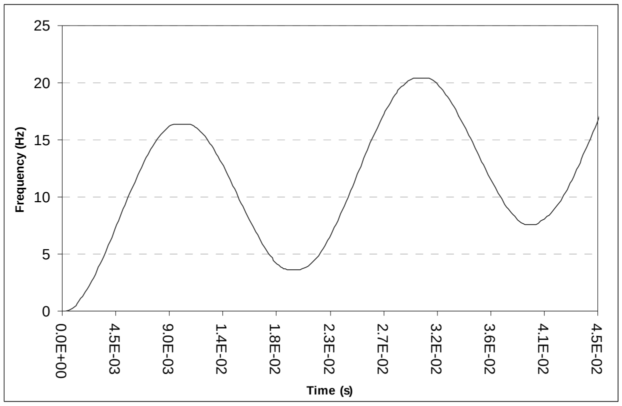

Figure 8. Integrated Faraday signal's converted frequency response with respect to time. Scaling factor is precisely defined in terms of Verdet constant and generalized amplification terms.

III. ACKNOWLEDGMENT

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IV. REFERENCES

[1] S. C. Rashleigh and R. Ulrich. 1979. Magneto-optic Current Sensing with Birefringent Fibers, Appl. Phys. Lett. 34(11).

[2] L. R. Veeser, G. I. Chandler, and G. W. Day. 1986. Fiber Optic Sensing of Pulsed Currents, Photonics: High Bandwidth Analog Application, Society of Optical Engineering 648.

[3] A.M. Smith. 1978. Appl. Opt. 17, 52.

[4] A. Papp and H. Harms. 1977. Appl. Opt. 16, 1315.

[5] H. Schneider, H. Harms, A. Papp, and H. Aulich. 1978. Appl. Opt. 17, 3035.

[6] D. H. Bailey and P. N. Swarztrauber. 1991. The Fractional Fourier Transform and Applications, Society for Industrial and Applied Mathematics Review, 33, 3.